SEM-4

(MATHEMATICS GENERAL)

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3) $\langle Z, +\rangle$ forms a group under addition. $H = \{3x \mid x \in Z\}$ $\subset Z$ forms a subgroup of Z.

Then the right cosets and left cosets determined by 0, 1, 2 are respectively

$H+0 = \{\cdots -9, -6, -3, 0, 3, 6, 9, \cdots\} = 0+H$

$H+1 = \{\cdots -8, -5, -2, 1, 4, 7, 10 \cdots\} = 1+H$

$H+2 = \{\cdots -7, -4, -1, 2, 5, 8, 11, \cdots\} = 2+H$

The other right cosets and left cosets determined by $a \in Z$ are equivalent to $H+0$, $H+1$, $H+2$ for $a = \pm 3, \pm 4, \cdots$

clearly, $Z = (H+0) \cup (H+1) \cup (H+2)$. Similarly, $Z = (0+H) \cup (1+H) \cup (2+H)$

Remarks: 1) Ha and aH are non-empty.

Since $e \in H \Rightarrow ea \in Ha \Rightarrow a \in Ha$. Thus Ha contains at least one element a. $Ha \neq \phi$. Similarly $aH \neq \phi$.

2) H is itself a right and left cosets of H, determined by the identity element e.

Since $He = \{he \mid h \in H\} = \{h \mid h \in H\} = H$

and $eH = \{eh \mid h \in H\} = \{h \mid h \in H\} = H$

3) In general, $Ha \neq aH$

In this case if the group be abelian $Ha = aH$ $\forall a \in G$ for some subgroup H of G.

Let $S = \{1, 2, 3\}$. Then $S_3 = \{p_0, p_1, p_2, p_3, p_4, p_5\}$. It forms a non-abelian symmetric set on S. It forms a non-abelian group under permutation composition.

Here $p_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$,

$p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

Let $H = \{p_0, p_3\}$. Then H forms a sub-group of S_3, since $p_3 \circ p_3 = p_0$

$$Hp_1 = \{p_1, p_4\}, \quad p_1 H = \{p_1, p_5\} \text{ so } Hp_1 \neq p_1 H.$$

Again, Let $H = \{p_0, p_1, p_2\}$. Then H forms a subgroup of S_3, since

$$p_1 p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = p_2$$

$$p_2 p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = p_1$$

$$p_1 p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = p_0$$

$$p_2 p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = p_0$$

Again $p_0 H = Hp_0 = H$, $p_3 H = Hp_3 = \{p_3, p_4, p_5\}$
$p_1 H = Hp_1 = H$, $p_4 H = Hp_4 = \{p_4, p_5, p_3\}$
$p_2 H = Hp_2 = H$, $p_5 H = Hp_5 = \{p_5, p_3, p_4\}$
Therefore, $pH = Hp \ \forall\, p \in S_3$

Thus, though S_3 is a non-abelian group, $pH = Hp \ \forall\, p \in S_3$.

Ex: 4 $\langle Z, + \rangle$ for a group under addition. Let $H = 5Z = \{5n \mid n \in Z\}$. Then $\langle H, + \rangle$ is a subgroup of the group $\langle Z, + \rangle$.

Now left cosets of H in Z are given by $n + H$ for all $n \in Z$ ie $n + 5Z$ for $n = 0, \pm 1, \pm 2, \ldots$

Any integer n is of the form $5m + r$, where $r = 0, 1, 2, 3, 4$. Hence $n + 5Z = 5m + r + 5Z = r + 5m + 5Z = r + 5Z$. Hence left cosets of $H = 5Z$ in Z are $0 + 5Z$, $1 + 5Z$, $2 + 5Z$, $3 + 5Z$, $4 + 5Z$. Similarly, we can find the right cosets.